A Qualitative Theory for Shape Representation

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Abstract

Shapes represent a very important way with which we perceive and reason about the world. In this paper we present an ordering information approach to the qualitative description of shapes. The shapes are described by a qualitative representation of angles, side lengths and describing the concavities and convexities of the boundary.

Keywords: Qualitative Spatial Reasoning, Qualitative Shape.

1. Introdution

Our environment is plenty of objects which can be described in terms of their shape. The shape of an object is the description of the properties of the boundary of the object. The boundary of the object is described by a set of points. A single point has neither dimension nor shape, but a one-dimensional curve has a shape that can be described.

A purely quantitative representation is when figures are described as mathematical functions of space coordinates. For instance a round disk can be described by the following mathematical function:

$$x^2+y^2=r^2.$$

For more complex shapes, it is general difficult to find a numerical function for the curve or surface describing the boundary of the figure. Piecewise interpolation methods are often used as a simplification. This means that the object to be described is approximated as consisting of many small parts, for instance straight lines or flat surfaces, for which it is possible to find numerical functions. The set of functions then make up the quantitative description of the shape of the object. An alternative quantitative representation is to approximate the shape of the object by the pixels it occupies. Depending on the resolution, this gives a more or a less coarse result, since some pixels may be only partially filled. Furthermore, the description of the shape may be quite different if it is rotated or translated within the grid.

In the artificial vision field it is necessary a high computational cost for image processing. Moreover, object recognition from image processing is an unsolved problem, i.e. it is not possible to distinguish the same chair from different points of view or a if it is partially hidden by using quantitative image processing.

Therefore, the definition and use of a theory for qualitative description of shape is important in the vision recognition field. The use of a qualitative theory for shape description and recognition will increase the efficiency in vision recognition because the recognition of a shape or an environment will be carried out by looking for only the distinguishing features and not analysing each pixel of the image.

A purely qualitative representation may describe shapes by linguistic terms, such as “round”, “straight” and so on. Figure 1 shows an example of a quantitative and a qualitative representation of a round disk.

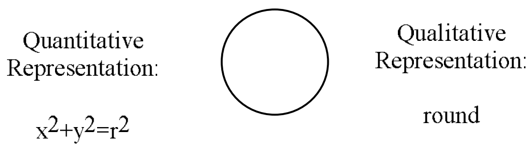


Figure 1. Examples of quantitative and qualitative representation od a round disk.

Most of the qualitative approaches to shape description can be classified as follows:

- Axial representations: these approaches are based on a description of the axes of an object, describing the shape qualitatively by reducing it to a “skeleton” or “axis”. The “axis” is a planar arc reflecting some global or local symmetry or regularity within the shape. The shape can be generated from the axis by moving a geometric figure (named “generator”) along the axis and sweeping out the boundary of the shape. The generator is a constant shape and keeps a specified point (i.e. its centre) but can change its size and its inclination with respect to the axis. Axial representations are qualitative since many different shapes can be generated from a given axis, using generators of different shape and varying sizes and angles. For two-dimensional shape generally either a circle or a straight line segment is used as a generator. Some works inside this group correspond to [19], [2], [17]. One important type of approaches classified as Axial representations are the so called Curvature Extrema approaches ([19], [17]), It is based on the idea which come from the fact that when asked to indicate the most salient points of a contour many people choose the points of extrema curvature, that means the points where the curve bends the most.
- Primitive-based representations: in these approaches complex objects are described as combinations of more primitive and simple objects. Primitive-based approaches in 3D shape representation describe an object in terms of solid primitives covering its volume. Basically primitive-based approaches can be classified into two schemes, ehich differ in the type of description they provide and the application field in which they are used. These schemes are:
  - Generalized cylinder and geon-based representations, which describe an object as a set of primitives plus a set of spatial connectivity relations among them ([1] and [12]). They provide qualitative description of an object and then the information embedded in such models can be used to distinguish between different objects but not to generate synthetic images of objects.
  - Constructive representations, which describe an object as the Boolean combination of primitive point sets ([22], [3] and [11]). These approaches have been used as quantitative description in CAD, where the primitives are specified in terms of numerical parameters, thus they can be used to generate a synthetic image of an object.
- Ordering- and Projection-based representation: in these approaches different aspects of the shape of an object are represented by either looking at it from different angles or by projecting it onto different axes ([18], [23], [13] and [4], [20], [7]). Most of these approaches are suitable for object recognition in image understanding.
- Topology and logic-based representations: these approaches rely on topology and/or logics in representing shapes ([6] and [21], [5]). In [21] the concept of convex-hull of a region is used to describe shapes. By convex-hull of a region they mean the smallest convex region of which it is a part. If one were to stretch an elastic membrane round a region then the convex-hull would be the whole of the region enclosed. They classify shapes as similar (belonging to the same class) if their convex-hull are similar.
- Cover-based representations: in these approaches the shape of an object is described by covering it with simple figures, as rectangles and spheres ([8]).

The theory proposed in this article can be classified as an ordering based representation due to the theory orders the vertices of the objects to give its description, and it has been described in order to work interactively with the reasoning process for robot navigation defined in [10], fact which explains the necessity of defining a new qualitative theory for qualitative shape description. Our robot navigation algorithm [10] is defined using Freksa and Zimmermann's orientation model ([14], [15], [24]) as basis. We want to apply this shape theory to object recognition in the same robot navigation problem. Therefore it is really interesting to us the integration of the same concepts used in the robot navigation algorithm into our model of shape representation. Moreover, the theory for qualitative description defined in this approach considers the qualitative length of the edges of the object as an important qualitative characteristic to consider, and this aspect has not been considered in other approaches.

2. The ordering information approach to Qualitative Shape Representation.

2.1. Basic assumptions

The 2-dimensional ordering relations are defined for points, as a consequence, shape description using ordering information will have to make use of some reference points. As reference points we understand that points which completely specify the boundary. For polygonal boundaries it is natural to choose the vertices as reference points. As up to now our interest is not in developing a theory for curve fitting we have restricted the theory for qualitative shape description to describing polygonal shapes. Boundaries will always be polygons linking the reference points in the order in which they are numbered.

The basic assumptions to the qualitative theory for shape representation are referred to the way in which the reference points are numbered. They are:

- The start of the reasoning process is always in the uppermost (left) corner (vertex) of the object.
- The vertices are numbered from the starting vertex in a counter clockwise way.
- The edges between two vertex are classified as concave or convex.
- The angles in each vertex are either right-angled, acute or obtuse.
- The relative length of each edge between two contiguous vertices is also defined using a length model for named lengths. The length model that we are going to define in inspired in the model by [16] and [9]. This model is explained in next section.

2.2. The length model

For named lengths it is necessary to define a reference system (which we will call Length Reference System (LRS)). The LRS has three components, LRS={UL,LAB,INT}, where UL refers to the unit of length used (mm, cm, m, …) which is context dependent. LAB refers to the set of qualitative lengths labels, the amount of which depend on the granularity level. INT refers to the intervals associated to each length label of LAB, which will describe the length label in terms of the UL.

Assuming that in a determined context we fix UL (whose value might change depending on the context) to ul, we are going to define as examples two different LRS at coarse and fine levels of granularity, respectively.

For the coarse LRS we define:

LAB_1={zero, small, normal, big}, and the corresponding $INT_1=\{[0,0],]0,ul/2],]ul/2,ul],]ul,\infty[\}$.

That means that for length "zero" we have a point.

For the fine LRS we define:

LAB_2={zero, very small, small, normal, big, very big}, and the corresponding $INT_2=\{[0,0],]0,ul/4],]ul/4,ul/2],]ul/2,ul],]ul,2ul],]2ul, \infty[\}$.

2.3. The Qualitative Shape Representation

The qualitative theory defined is going to be used during the autonomous robot navigation defined in [10] to reason about the shape of the obstacles and the environment.

The qualitative shape representation theory is defined using as tool the Freksa and Zimmermann's orientation Reference System [14,15] augmented by a circle as it is described in [24]. In Freksa and Zimmermann's orientation model the space is divided into qualitative regions by means of a

Reference System (RS). The RS is formed by an oriented line determined by two reference objects (from a point, a, to another point, b) –which defines the left/right dichotomy-, the perpendicular point by b –which defines the first front/back dichotomy-, and the perpendicular line by a- which establishes the second front/back dichotomy and defines a fine division of the space in the back part of the RS. If only the first/back dichotomy is considered (the RS is then called coarse) the space is divided into 9 qualitative regions. If both perpendicular lines are taken into account (the RS in then called fine) the space is divided into 15 qualitative regions. In our approach we focus our attention in the fine RS (figure 2a). An iconical representation of the fine RS and the names of the regions are shown in figure 2b). The information which can be represented by this RS is the qualitative orientation of a point object, c, with respect to the RS formed by the point objects a and b, that is, c wrt ab.

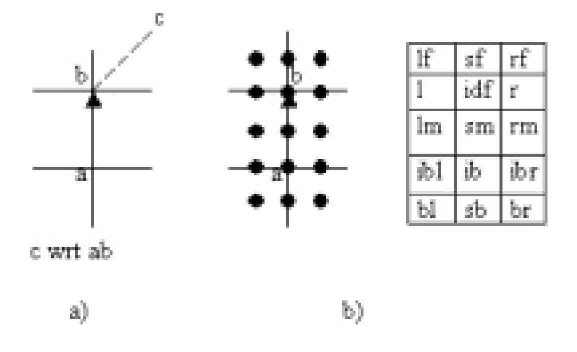


Figure 2. a) The fine RS for qualitative orientation; b) the iconical representation in which letters corresponds to l: left, r: right, f: front, s: straight, m: middle, b: back,, i: identical.

For the description of the angle type, we are going to use the augmented orientation Reference System by Zimmerman and Freksa [24]. This RS is defined by an oriented line from a to b, plus the two perpendicular lines by b and a respectively, and a circle with diameter ab (figure 3).

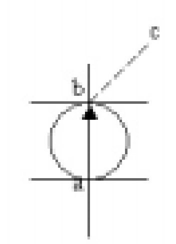


Figure 3. The fine orientation RS augmented by a circle with diameter ab.

The central idea of the qualitative shape representation consists in given three reference points i, j, k, which are consecutive due to the numeration given in a counter clockwise sense of the vertices of the object, the qualitative description of the reference point j is determined by positioning the orientation RS between the points i and k as figure 4 shows. In figure 3 i is the vertex 1, j is the vertex 2 and k is the vertex 3. In this figure the orientation RS is placed from 1 to 3.

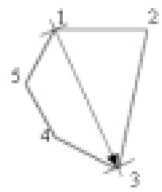


Figure 4. Example of a shape figure in which we are determining the qualitative description of vertex 2 using vertex 1 and 3 as the reference vertex for the orientation Freksa and Zimmermann's RS.

The qualitative description of the reference point j is given by a set of three elements (triple) <Aj,Cj,Lj> where Aj means the angle for the reference point j, Cj means the type of convexity of point j and Lj means the length of the edge associated to reference point j (edge formed by edges i and j), where:

$A_j \in$ {right-angled, acute, obtuse};

$C_j \in$ {convex, concave} and

L_j belongs to the Length Reference System used for a particular context, for instance Lj belongs to the fine LRS defined in section 2.2.

The convexity of the point j is determined by the orientation RS as follows: if the reference point j remains on the left dichotomy created by the oriented line from i to k then the point j is a convex vertex. That means that the point j with respect to the Reference System formed by ik (j wrt ik) is on the left- front or left or left-medium or identical-back left or back-left. Otherwise if the point j remains on the right dichotomy created by the oriented line from i to k then the point j is concave, which means that j wrt ik is right-front or right or right-medium, or identical-back right or back-right. As a vertex appears when the orientation of the edge changes then it is not possible that the reference point j remains exactly over the oriented line from i to k. Formally, if Vj means vertex j we can formulate:

If Vj wrt ViVk ∈ [lf, l, lm, ibl, bl] then Vj is convex.

If Vj wrt ViVk ∈ [rf, r, rm, ibr, br] then Vj in concave.

The length calculated in the reference point j is the length of the edge from the point i to the point j using the LRS chosen depending on the context. Therefore the label for this point is assigned in function of the set of intervals of acceptance (INT) defined in the LRS. Therefore it is inferred as, being the length of the segment formed by vertex Vi and Vj (called length(Vj)) calculated by |ViVj|, then the qualitative label for named length is determined, using the fine RS given in this article, as:

If length(Vj) Î INTh, where hÎ{0,1,2,3,4,5} and corresponds to the six intervals defined in the set INT of the coarse LRS, that is INT0 = [0,0], INT1=]0,ul/2], …., INT5=]2ul, ¥[, then the vertex Vj has as qualitative named length the label LABh, where h Î{0,1,2,3,4,5}, which is the label associated to the interval INTh, that is LAB0=zero, LAB1=very small, … and LAB5=very big.

Finally the qualitative description of an angle is determined using the augmented orientation RS and some topological concepts as boundary, interior and exterior of an entity. Therefore, in order to understand how the angle of a vertex is determined we need to give the definitions of such topological concepts.

Definition 1. The boundary of an entity h, called δh is defined as:

- We consider the boundary of a point-like entity to be always empty.
- The boundary of a linear entity is the empty set in the case of a circular line, or the 2 distinct endpoints otherwise.
- The boundary of an area is the circular line consisting of all the accumulation points of the area.

Definition 2. The interior of an entity h, called h^o is defined as h^o=h-δh.

Definition 3. The exterior of an entity h, called h- is defined as h-=$\Re^2$-h.

In summary, if the reference point j remains exactly in the boundary of the circle of the augmented orientation RS, then the vertex j is right-angled, if j remains in the exterior of the circle then j is acute and if j remains in the interior of the circle then the vertex j is obtuse. Formally, if the circle of the augmented orientation RS with a diameter of ViVk is denoted as Cik, then the angle of the Vertex j (Vj) is calculated using the following algorithm:

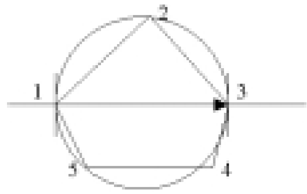
If $V_j \cap \delta C_{ik} \neq \varnothing$ then V_j is right- angled,

Else if $V_j \cap C_{ik}^o \neq \varnothing$ then V_j is obtuse

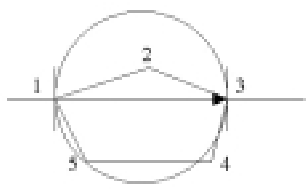
Otherwise V_j is acute

The part of the "otherwise" of the above algorithm occurs when $V_j \cap C_{ik}^- \neq \varnothing$.

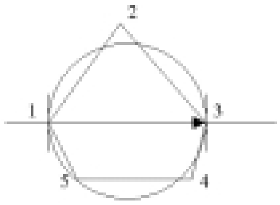
Next figure shows a graphical example for each of these cases (figure 5).



a) Right-angled angle



b) Obtuse angle



c) Acute Angle

Figure 5. Examples of determination of the angle of vertex 2, using the augmented RS formed using vertices 1 and 3 as reference vertices; a) for a right – angled angle; b) for an obtuse angle and c) for an acute angle.

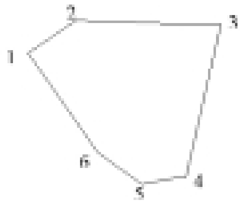
2.4. The Complete Description of a Shape

Given a polygonal shape its complete description is defined as a set of triples of the type <Ai,Ci,Li>. The number of triples describing the shape is equal to the number of reference points (vertices) of the shape. Formally, if we denote as QualShape(A) the Qualitative description of the Shape A, then it is defined as:

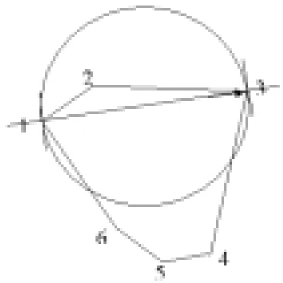
QualShape(A)=[<A1,C1,L1>,….<An,Cn,Ln>],
where n= number of vertices of the shape A.

Therefore, in order to describe completely a shape we have to repeat the process described in section 2.3, beginning by the vertex numbered by 1 (uppermost left vertex or corner), then we will determine the qualitative representation of vertex 2, until the vertex 1 is characterised. That means that we will use as reference vertices for the orientation RS the vertex n (last vertex of the shape) and vertex 2.

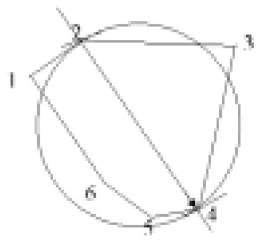
Figure 6 shows an example of how the qualitative representation of a shape is constructed.



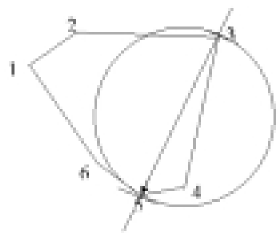
a) Original Shape with the vertices numbered



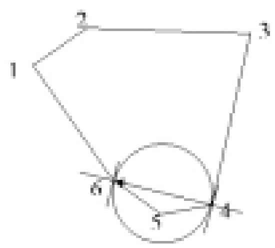
b) First step, determination of qualitative shape of vertex 2.



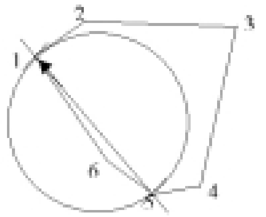
c) Second step, determination of qualitative shape of vertex 3.



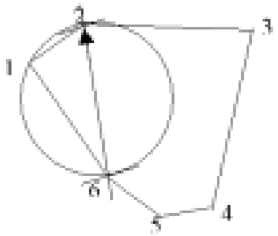
d) Third step, determination of qualitative shape of vertex 4.



e) Forth step, determination of the qualitative shape of vertex 5.



f) Fifth step, determination of the qualitative shape of vertex 6.



g) Sixth step, determination of the qualitative shape of vertex 1.

Figure 6. Sequence of steps to get the complete shape representation of the shape depicted in a).

In each step of the sequence of figure 5 a triple of triple <A_i, C_i, L_i> is defined. In this example, taken as LRF the coarse reference system we will get next triples for each vertex:

Vertex 2: <obtuse, convex, small>

Vertex 3: <acute, convex, normal>

Vertex 4: <obtuse, convex, big>

Vertex 5: <obtuse, convex, small>

Vertex 6: <obtuse, convex, small>

Vertex 1: <right – angled, convex, normal>.

Finally the complete description of the shape is described using the description of each vertex from the first one to the last one as follows:

[<right–angled, convex, normal>, <obtuse, convex, small>,<acute, convex, normal>, <obtuse, convex, big>, <obtuse, convex, small>, <obtuse, convex, small>].

3. Applications

The qualitative shape representation presented in this paper will play an important role in a robot navigation task, because the navigation involves reasoning with regions with a shape.

Therefore, the theory described here could be used to describe the shape of the objects or obstacles that the robot can find during its movement and it also can describe the environment (regions) in which the robot is. If the qualitative navigation strategy of the robot organizes information about the visual order of landmarks, and we assume that the robot is able to see and identify landmarks from each position, then the robot will be able to get a qualitative representation of the shape of the part of the region that it is observing, formed by 3 landmarks applying the calculus here presented. Then, applying a qualitative shape matching method, it could know if there is an obstacle to avoid or if it is the goal region to which it is going, then it is describing the environment.

Finally, the way in which the theory has been defined makes the theory completely compatible with the integrated models of orientation, topology, length and path planning defined in [9,10]. Thus, the theory can be used jointly with these models to help into the qualitative spatial reasoning strategy for robot path planning defined in [10].

4. Conclusions and Future Work

A straightforward Qualitative Theory of Shape description is defined in this article. It will allow us to reason about shape in a qualitative way as human beings do. Moreover, most of the qualitative approaches developed nowadays are used for reasoning about object position, and the theory presented here allow us to use the same method to reason about position and shape. The theory proposed here is a example of ordering information approaches and it provides a simple example for representing quadrilateral shapes. The interest of ordering information for shape description relies in the fact that it is less constrained than metrical information but more constrained than topological information, which will not allow us to determine the convexity or concavity of the shape, neither the length of edges, nor the angle types.

Finally the proposed theory can be used to determine the shape of the objects (obstacles) and the shape of the environment in which a robot is navigating.

On the other hand, as future work we are interested in the development of a Qualitative Shape algebra, defining the operations needed to construct new forms shapes from a set of given shapes, that is, we should define operations such as union, intersection and difference of shapes. The idea is to define these operations in a topological way using the concepts of boundary, interior, exterior and dimension of the shapes. Moreover, next step is to use the qualitative algebra for qualitative shape matching and recognition tasks. In order to do this, we will need to describe some relevant properties of the shapes which could help us during the matching task. These properties could be properties as symmetry, alternation or iteration of some parts of the shape inside the shape itself.

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References

[1] Biederman, I., Recognition-by-Components: A theory of Human Image Understanding, *Psychol. Rev.,* 94, 2: 115-147, 1987.

[2] Brady, M., *Criteria for Representations of Shape,* Human and Machine Vision, 1983.

[3] Brisson, E., Representing Geometric Structures in d-dimensions: Topology and Order, *Proceedings 5th ACM Symposium on Computational Geometry,* Saarbruchen, 218-227, 1989.

[4] Chen, S. And Freeman, H., Computing characteristic views of quadratic-surfaced solids, *Proc. 10th Int. Conf. On Pattern Recognition,* 1990.

[5] Clementini, E. and Di Felice, P., A global framework for qualitative shape description. *GeoInformatica* 1(1):1-17, 1997.

[6] Cohn, A.G., A Hierarchical Representation of Qualitative Shape based on Connection and Convexity, *Proceedings COSIT'95,* Springer-Verlag, 311-326, 1995

[7] Damski, J. C. and Gero, J. S., A logic-based framework for shape representation, *Computer-Aided Design* 28(3):169-181, 1996.

[8] Del Pobil, A.P., Serna, M.A., Spatial Representation and Motion Planning, *Lecture Notes in Computer Science no. 1014,* Springer-Verlag, 1995.

[9] Escrig, M. T and Toledo, F., *Qualitative Spatial Reasoning: Theory and Practice,* IOS PRESS, 1998.

[10] Felip, P., Escrig, M.T., Qualitative Spatial Reasoning for Robot Path Planning, in press for the CCIA'2003.

[11] Ferrucci, V. and Paoluzzi, A. Extrusion and Boundary Evaluation for Multidimensional Polyhedra, *Computer Aided Design,* 23, 1: 40-50, 1991.

[12] Flynn , P.J. and Jain, A.K., CAD-based Computer Vision: From CAD Models to Relational Graphs, *IEEE Transaction on P.A.M.I.,* 13:114-132, 1991.

[13] Freeman, H. And Chakravarty, I., The use of characteristic views in the recognition of three-dimensional objects, *Pattern Recognition in Practice,* 1980.

[14] Freksa, C., Conceptual Neighborhood and its role in temporal and spatial reasoning, *Proceedings of the IMACS WorkShop on Decision Support Systems and Qualitative Reasoning,* 181-187, 1991.

[15] Freksa, C., Using Orientation Information for Qualitative Spatial Reasoning, *in A.U. Frank, I. Campari, and U. Formentini (eds), Theories and Methods of Spatio-Temporal Reasoning in Geographic Space, Proc. Int. Conf. GIS- From Space to Territory, volume 639 of Lecture Notes in Computer Sciences*, 162-178, 1992.

[16] Hernández, D., Clementini, E. and Di Felice, P., Qualitative Distances, *in Frank and Kuhn (eds.) Spatial Information Theory. A theoretical Basis for GIS. International Conference COSIT'95, volume 988 of Lecture Notes in Computer Science,* Springer, 1995.

[17] Richards W., Hoffman D., Codon constraints on closed 2D shapes, *Computer Vision, graphics and image processing*, 31, 265-281, 1985.

[18] Jungert, E. Symbolic spatial reasoning on object shapes for qualitative matching, *Spatial Information Theory,* Springer Verlag, Berlin, 444-462, 1994.

[19] Leyton, M., A process-grammar for shape, *Artificial Intelligence*, 34:213-247, 1988.

[20] Park, S.H. and Gero, J. S. Qualitative representation and reasoning about shapes, *in*

Gero, J. S. and Tversky, B. (eds), *Visual and Spatial Reasoning in Design* , pp. 55-68, 1999.

[21] Randell, D.A., Cui, Z. And Cohn, A.G., A spatial logic based on regions and connection, *Proceedings 3rd Int. Conf. On Knowledge Representation and Reasoning,* Morgan Kaufmann, 1992.

[22] Requicha, A.A.G., Representations of Rigid Solids: Theory, Methods, and Systems, *ACM Computing Surveys,* 12, 4:437-464, 1980.

[23] Schlieder, Ch. Qualitative Shape Representation, *Spatial conceptual models for geographic objects with undertermined boundaries*, Taylor and Francis, London, 123-140, 1996.

[24] Zimmermann, K. and Freksa, C., Qualitative Spatial Reasoning Using Orientation, Distance, and Path Knowledge, *Applied Intelligence 6,*49-58, 1996.